

Dirichlet Higgs as radion stabilizer in warped compactification

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Abstract

We study implications of generalized non-zero Dirichlet boundary condition along with the ordinary Neumann one on a bulk scalar in the Randall-Sundrum warped compactification. First we show profiles of vacuum expectation value of the scalar under the general boundary conditions. We also investigate Goldberger-Wise mechanism in several setups with the general boundary conditions of the bulk scalar field and find that the mechanism can work under non-zero Dirichlet boundary conditions with appropriate vacuum expectation values. Especially, we show that $SU(2)_R$ triplet Higgs in the bulk left-right symmetric model with custodial symmetry can be identified with the Goldberger-Wise scalar.

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1 Introduction

One of the main targets of the CERN Large Hadron Collider (LHC) is the discovery of an evidence of extra-dimension(s) as well as the Higgs particle. There are extra dimensional alternatives to the ordinary electroweak symmetry breaking (EWSB) mechanism in the Standard Model (SM), such as the gauge-Higgs unification (GHU) [1, 2, 3, 4], the little Higgs [5], the Higgsless [6], and the Dirichlet Higgs [7] models¹, and so on. Possible approaches to address the Higgs mass hierarchy problem are large extra-dimension scenario [14, 15] and the Randall-Sundrum (RS) model [16] in addition to supersymmetry.

The extra-dimensional models can also give phenomenologically interesting features and predictions,² for example, a candidate for dark matter (DM) from the Universal Extra-Dimensions (UED) model [18] and deviations of couplings of the Higgs in the context of GHU scenario [19, 20], brane localized Higgs potential models [21, 22], and Dirichlet Higgs model [7, 23, 24]³. Constructing realistic models in the warped five-dimensional space-time proposed by Randall and Sundrum is still an interesting issue. After this proposal, Goldberger and Wise (GW) presented a mechanism for stabilizing the size of the extra-dimension in RS scenario [26]. In the GW mechanism, the potential for the radion, which determines the size of radius of the extra-dimension can be generated by a bulk scalar field with quartic couplings of brane localized potentials. As a result, the potential minimum gives a favored compactification scale to solve the hierarchy problem. Then a simple extension of the SM in the bulk of the warped extra-dimension [27] and a model with custodial symmetry [28] have been proposed.

In this paper, we will focus on a bulk scalar field theory under general boundary conditions (BCs) on the warped five-dimension. We analyze profiles of vacuum expectation value (VEV) of the scalar under the general BCs. We also investigate GW mechanism with the general BCs of the bulk scalar field. We point out that the mechanism can work under non-zero Dirichlet BCs that give appropriate VEVs. We also consider a scenario that a bulk Higgs field plays a role of GW mechanism. Especially, we will show that $SU(2)_R$ triplet Higgs in a model with custodial symmetry can be identified with the bulk scalar of GW mechanism.

This paper is organized as follows. In section 2, we study behaviors of bulk scalar field under possible four BCs on the warped extra-dimension. In section 3, we investigate the GW mechanism under the BCs in several setups. We will try to identify the bulk

¹See also Refs. [8, 9], [10, 11], [12, 13], and references therein for nice reviews of the GHU, little Higgs, and Higgsless models, respectively.

²First proposal of TeV scale compactification is made in [17].

³An implementation of such deviations to flavor physics has been discussed in [25], which can also give a DM candidate.

scalar in the GW mechanism as the Higgs in the bulk SM or $SU(2)_R$ triplet Higgs in a model with custodial symmetry. The discussions in this section will be proceeded with some reviews of related important models and mechanisms. The section 4 is devoted to summary. Relatively technical discussions are shown in Appendices.

2 Bulk scalar in warped extra-dimension

In this section, we study a bulk scalar field theory on the warped extra-dimensional space-time proposed by Randall and Sundrum, and clarify the wave function profiles of classical mode of the scalar under four BCs.

We start with the following action of a bulk field, Φ ,

$$S = \int d^5x \sqrt{-G} [-G^{MN} (\partial_M \Phi^\dagger) (\partial_N \Phi) - \mathcal{V}(|\Phi|^2)], \quad (1)$$

where $x^M = (x^\mu, y) = (x^0, \dots, x^5)$, $y = x^5$. For simplicity, we assume that the potentials solely depend on $|\Phi|^2$ so that potentials can be written as $\mathcal{V}(|\Phi|^2)$. The metric is given by

$$G_{MN} dx^M dx^N = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad \text{and} \quad G^{MN} \partial_M \partial_N = e^{2\sigma} \eta_{\mu\nu} \partial_\mu \partial_\nu + \partial_y^2, \quad (2)$$

where

$$\sigma \equiv k|y|, \quad \sigma' = k\epsilon(y), \quad \sigma'' = 2k[\delta(y) - \delta(y - L)], \quad \eta_{\mu\nu} \equiv \text{diag}\{-1, 1, 1, 1\}. \quad (3)$$

The $\epsilon(y)$ is a kind of sign function defined by $\epsilon(\pm|y|) = \pm 1$ and $\epsilon(0, L) = 0$. The k is the brane tension, which is related to the bulk energy density (cosmological constant Λ) and the brane potential energy by

$$k \equiv \pm \sqrt{\frac{-\Lambda}{6M_5^3}} = \frac{V_{UV}}{6M_5^3} = -\frac{V_{IR}}{6M_5^3}, \quad (4)$$

where M_5 is the Planck mass in five-dimensions. The stable and flat configurations of the branes can be realized when the relations (4) is satisfied. By utilizing the above descriptions, the action (1) can be rewritten by

$$S = \int d^4x \int_0^L dy e^{-4\sigma} [-e^{2\sigma} |\partial_\mu \Phi|^2 - |\partial_y \Phi|^2 - \mathcal{V}]. \quad (5)$$

We define the action on a line segment as $0 \leq y \leq L$. When we write the bulk scalar field as

$$\Phi = \frac{\Phi_R + i\Phi_I}{\sqrt{2}}, \quad (6)$$

we obtain

$$\frac{\partial \mathcal{V}}{\partial \Phi_X} = \Phi_X \mathcal{V}', \quad \frac{\partial^2 \mathcal{V}}{\partial \Phi_X^2} = \mathcal{V}' + \Phi_X^2 \mathcal{V}'', \quad \frac{\partial^2 \mathcal{V}}{\partial \Phi_R \partial \Phi_I} = \Phi_R \Phi_I \mathcal{V}'', \quad (7)$$

where X stands for R and I , and we have written $\mathcal{V}' = d\mathcal{V}/d(|\Phi|^2)$ etc.. The variation of the action is given by

$$\begin{aligned} \delta S = \int d^4x \int_0^L dy e^{-4\sigma} & \left[\delta \Phi_X \left(\mathcal{P} \Phi_X - \frac{\partial \mathcal{V}}{\partial \Phi_X} \right) \right. \\ & \left. + \delta(y) \delta \Phi_X (+\partial_y \Phi_X) + \delta(y-L) \delta \Phi_X (-\partial_y \Phi_X) \right], \end{aligned} \quad (8)$$

where we define as $\mathcal{P} = e^{2\sigma} \square + e^{4\sigma} \partial_y e^{-4\sigma} \partial_y$. The VEV of the scalar field is determined by the action principle, $\delta S = 0$, that is,

$$\mathcal{P} \Phi_X - \frac{\partial \mathcal{V}}{\partial \Phi_X} = 0, \quad (9)$$

while the BC at $y = 0$ and L reads either Dirichlet

$$\delta \Phi_X|_{y=\eta} = 0 \quad (10)$$

or Neumann

$$\pm \partial_y \Phi_X|_{y=\eta} = 0, \quad (11)$$

where signs above and below are for $\eta = 0$ and L , respectively⁴. We can have four choices of combination of Dirichlet and Neumann BCs at $y = 0$ and L , namely (D, D) , (D, N) , (N, D) , and (N, N) . Different choice of BC corresponds to different choice of the theory. Once the theory is fixed, one of the four conditions is determined.

We study behaviors of the bulk scalar field on the warped five dimension by utilizing the background field method, separating the field into classical and quantum fluctuation parts:

$$\Phi(x, y) = \Phi^c(x, y) + \phi^q(x, y). \quad (12)$$

The configuration of the classical field obeys the EOM (9),

$$\mathcal{P} \Phi_X^c - \frac{\partial \mathcal{V}^c}{\partial \Phi_X^c} = 0, \quad (13)$$

with either the Dirichlet BC

$$\delta \Phi_X^c|_{y=\eta} = 0, \quad (14)$$

⁴If one considers a case with brane localized potential, the Neumann type BCs are changed. The formulation for the case with brane localized potential is given in the Appendix A. For our purpose of this paper, it is enough to discuss in the absence of the brane potentials and main results are not modified.

or the Neumann BC

$$\pm \partial_y \Phi_X^c|_{y=\eta} = 0, \quad (15)$$

at each brane⁵. Here and hereafter, we use the following shorthand notation,

$$\frac{\partial V^c}{\partial \Phi}(x, y) \equiv \frac{\partial V}{\partial \Phi} \Big|_{\Phi=\Phi^c(x, y)}, \quad \frac{\partial^2 V^c}{\partial \Phi^2}(x, y) \equiv \frac{\partial^2 V}{\partial \Phi^2} \Big|_{\Phi=\Phi^c(x, y)}, \quad (16)$$

etc..

For simplicity, we take the bulk potential as

$$\mathcal{V} = m^2 |\Phi|^2 = \frac{m^2}{2} (\Phi_R^2 + \Phi_I^2). \quad (17)$$

In this case, the EOM (9) can be written down as

$$(\partial_y^2 - 4k\partial_y - m^2)\Phi_X^c = 0. \quad (18)$$

The solution of this equation is given by

$$\Phi_X^c(z) = Az^{\nu+2} + Bz^{-(\nu-2)}, \quad (19)$$

where $\nu \equiv \sqrt{4 + m^2/k^2}$ and $z \equiv e^\sigma$.

Next, let us study the profile of quantum fluctuation of the scalar field. We separate the field into the classical field and quantum fluctuation as

$$\Phi(x, y) = \frac{1}{\sqrt{2}} [v(y) + \phi(x, y) + i\chi(x, y)], \quad (20)$$

$$= \frac{1}{\sqrt{2}} \left[v(y) + \sum_{n=0}^{\infty} f_n^\phi(y) \phi_n(x) + i \sum_{n=0}^{\infty} f_n^\chi(y) \chi_n^q(x) \right], \quad (21)$$

where we took $\Phi_R = v(y)$ and $\Phi_I = 0$, and the Kaluza-Klein (KK) expansions are taken for $\phi(x, y)$ and $\chi(x, y)$. We put separation (20) into (5) and expand up to the quadratic terms of the field ϕ and χ as⁶

$$\begin{aligned} S(\phi, \chi) = & \int d^4x \int_0^L dy e^{-4\sigma} \left[\frac{1}{2} \phi \left(e^{2\sigma} \square + e^{4\sigma} \partial_y e^{-4\sigma} \partial_y - \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_R^2} \right) \phi \right. \\ & + \frac{1}{2} \chi \left(e^{2\sigma} \square + e^{4\sigma} \partial_y e^{-4\sigma} \partial_y - \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_I^2} \right) \chi \\ & \left. - \frac{\delta(y)}{2} (-\phi \partial_y \phi - \chi \partial_y \chi) - \frac{\delta(y-L)}{2} (+\phi \partial_y \phi + \chi \partial_y \chi) \right]. \quad (22) \end{aligned}$$

⁵The additional effects induced by the brane terms also change the VEV and quantum field wavefunction profiles, which lead to interesting phenomenological consequences [21, 22].

⁶The derivation of the action with brane localized potentials is given in Appendix A.

This corresponds to the bulk action for ϕ . By utilizing KK expansion, the KK equation is given by⁷

$$e^{-4\sigma} \left(\partial_y^2 - 4k\partial_y - \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_R^2} \right) f_n^\phi(y) = -\mu_{\phi_n}^2 f_n^\phi(y). \quad (23)$$

The general Dirichlet and Neumann BCs are

$$f_n^\phi(y)|_{y=\eta} = 0, \quad (24)$$

and

$$\pm \partial_y f_n(y)|_{y=\eta} = 0, \quad (25)$$

respectively. In this setup, we investigate the profile of bulk scalar under the above four BCs.

2.1 (D, D) case

First, we study a case in which both BCs on the $y = 0$ and $y = L$ branes, which correspond to $z = 1$ and $z = e^{kL} \equiv z_L$ ones respectively, are the Dirichlet type BCs. The most general form of the Dirichlet BC is $\delta\Phi|_{z=\xi} = 0$ and

$$v(1) = v_1, \quad v(z_L) = v_2, \quad (26)$$

where ξ is taken as 1 and z_L . These BCs can be rewritten as

$$A + B = v_1, \quad Az_L^{\nu+2} + Bz_L^{-(\nu-2)} = v_2, \quad (27)$$

by utilizing the general solution (19) of the EOM. The (27) lead to

$$A = -\frac{v_1 z_L^{-(\nu-2)} - v_2}{z_L^{\nu+2} - z_L^{-(\nu-2)}}, \quad B = \frac{v_1 z_L^{\nu+2} - v_2}{z_L^{\nu+2} - z_L^{-(\nu-2)}}. \quad (28)$$

Therefore, under these BCs, we obtain the VEV profile as

$$v(z) = -\frac{v_1 z_L^{-(\nu-2)} - v_2}{z_L^{\nu+2} - z_L^{-(\nu-2)}} z^{\nu+2} + \frac{v_1 z_L^{\nu+2} - v_2}{z_L^{\nu+2} - z_L^{-(\nu-2)}} z^{-(\nu-2)}. \quad (29)$$

A typical profile is shown in Tab. 1. It is seen that the VEV profile localizes toward to the IR brane. In order to solve the hierarchy problem in the RS background, the magnitude of z_L becomes $\mathcal{O}(10^{15})$.

⁷We focus only on the ϕ field throughout this paper unless it is needed to distinguish between the ϕ and χ fields.

2.2 (D, N) case

Next, let us consider the (D, N) case. These BCs can be described as

$$v(1) = v_1, \quad \partial_z v(z)|_{z=z_L} = 0. \quad (30)$$

Then, they are written down by

$$A + B = v_1, \quad A(\nu + 2)z_L^{\nu+2} - B(\nu - 2)z_L^{-(\nu-2)} = 0. \quad (31)$$

These lead to

$$v(z) = \frac{v_1(\nu - 2)z_L^{-(\nu-2)}}{(\nu + 2)z_L^{\nu+2} + (\nu - 2)z_L^{-(\nu-2)}}z^{\nu+2} + \frac{v_1(\nu + 2)z_L^{\nu+2}}{(\nu + 2)z_L^{\nu+2} + (\nu - 2)z_L^{-(\nu-2)}}z^{-(\nu-2)}. \quad (32)$$

The profile is illustrated in Tab. 1.

2.3 (N, D) case

In (N, D) BC case, the BCs are

$$\partial_z v(z)|_{z=1} = 0, \quad v(z_L) = v_2. \quad (33)$$

And they can be written down as

$$A(\nu + 2) - B(\nu - 2) = 0, \quad Az_L^{\nu+2} + Bz_L^{-(\nu-2)} = v_2. \quad (34)$$

These lead to

$$v(z) = \frac{(\nu - 2)v_2}{(\nu - 2)z_L^{\nu+2} + (\nu + 2)z_L^{-(\nu-2)}}z^{\nu+2} + \frac{(\nu + 2)v_2}{(\nu - 2)z_L^{\nu+2} + (\nu + 2)z_L^{-(\nu-2)}}z^{-(\nu-2)}. \quad (35)$$

The profile is shown in Tab. 1.

2.4 (N, N) case

Finally, we discuss the (N, N) BC case. The BCs are

$$\partial_z v(z)|_{z=1} = 0, \quad \partial_z v(z)|_{z=z_L} = 0, \quad (36)$$

and they are written down as

$$A[(\nu + 2) - B(\nu - 2)] = 0, \quad A(\nu + 2)z_L^{\nu+2} - B(\nu - 2)z_L^{-(\nu-2)} = 0. \quad (37)$$

We find that there is no solution to satisfy the above BCs except for a trivial one, $(A, B) = (0, 0)$, which might not have physical interests in any phenomenological models. That

does not depend on whether the brane localized potentials exist or not, that is, there is no solution, which is consistent with the BC, if the brane localized potentials exist. Therefore, it is not trivial that there is a viable VEV of a bulk scalar field satisfying the (N, N) type BCs in any phenomenological models, which would generally have brane localized interactions from radiative corrections.

In this section, we formulated a scalar field theory under four BCs on a warped five-dimensional background. Then a VEV profile of the bulk scalar field was given by both analytic and numerical computations. It is straightforward to extend the above discussions to a higher extra-dimensional background. In the next section, we study some applications of role of bulk scalar field in warped five-dimensional models.

3 Warped five-dimensional models with bulk scalar

We investigate some applications of role of bulk scalar field, whose VEV profile is presented in the previous section, in warped five-dimensional models. Especially, following applications are (re)considered, (i) realizations of the GW mechanism under general BCs discussed in the previous section, (ii) the bulk SM Higgs as the GW scalar, and (iii) a triplet Higgs under additional $SU(2)_R$ symmetry as the GW scalar.

3.1 GW mechanism and general BCs

In this section, we discuss the GW mechanism [26], where a bulk scalar plays an important role to stabilize the radion. We give a short review of this mechanism at first.

Goldberger and Wise proposed a mechanism for stabilizing the size of extra-dimension in the warped space. The GW mechanism starts with the bulk and brane actions for a bulk scalar field given in (64), (17), and (67). The VEV of the bulk scalar field can be classically obtained by solving the EOM, and its general solution is given in (19). The unknown coefficients A and B are determined by imposing BCs as we performed in the section 2.1-2.4. In the GW mechanism, the (N, N) type BCs have been taken. And the mechanism includes the brane localized potentials. Therefore, the BCs can be written down as

$$k[A(\nu + 2) - B(\nu - 2)] - \lambda_0(A + B)[(A + B)^2 - v_0^2] = 0, \quad (38)$$

$$\begin{aligned} & k[A(\nu + 2)z_L^{\nu+2} - B(\nu - 2)z_L^{-(\nu-2)}] \\ & + \lambda_L(Az_L^{\nu+2} + Bz_L^{-(\nu-2)})[(Az_L^{\nu+2} + Bz_L^{-(\nu-2)})^2 - v_L^2] = 0. \end{aligned} \quad (39)$$

However, the mechanism is considered in a case where the boundary quartic couplings λ_0 and λ_L are large. Here, one should note that the (N, N) BCs in the limit of large boundary

couplings with the potential (17) and (67) are equivalent to the (D, D) ones given in (26) if we take $v_1 = v_0$ and $v_2 = v_L$. Since A and B in the GW mechanism with the large coupling limits and in expansion of z_L^{-1} are actually given by

$$A_{\text{GW}} \simeq -v_0 z_L^{-2\nu} + v_L z_L^{-(\nu+2)}, \quad (40)$$

$$B_{\text{GW}} \simeq v_0(1 + z_L^{-2\nu}) - v_L z_L^{-(\nu+2)}, \quad (41)$$

it is easily seen that A and B given in (28) are $A \simeq A_{\text{GW}}$ and $B \simeq B_{\text{GW}}$ under the limits and replacements of $v_1 = v_0$ and $v_2 = v_L$. The above correspondence from the (N, N) BCs with large boundary couplings to (D, D) can be also simply understood in terms of the vacuum structure for an effective four-dimensional potential. The effective potential in four dimensions are written down by putting (19) back into (64) and integrating over the extra-dimension as

$$\begin{aligned} V_{\text{eff}} = & k(\nu + 2)A^2(z_L^{2\nu} - 1) + k(\nu - 2)B^2(1 - z_L^{-2\nu}) \\ & + \lambda_L z_L^{-4}[\Phi^c(z_L)^2 - v_L^2]^2 + \lambda_0[\Phi^c(1)^2 - v_0^2]^2. \end{aligned} \quad (42)$$

We find that $\Phi^c(1) = v_0$ and $\Phi^c(z_L) = v_L$ are energetically favored at the large boundary coupling limit, and thus, these solutions just correspond to the (D, D) BCs in (26) with $v_1 = v_0$ and $v_2 = v_L$.

In [26], it was assumed for simplicity that

$$\frac{m}{k} \ll 1, \quad (43)$$

so that

$$\nu = 2 + \epsilon \quad \text{with} \quad \epsilon \simeq \frac{m^2}{4k^2}. \quad (44)$$

Under this assumption, the effective potential becomes

$$V_{\text{eff}} = k\epsilon v_0^2 + 4k z_L^{-4}(v_L - v_0 z_L^{-\epsilon})^2 \left(1 + \frac{\epsilon}{4}\right) - k\epsilon v_0 z_L^{-(4+\epsilon)}(2v_L - v_0 z_L^{-\epsilon}). \quad (45)$$

For the purpose of moduli stabilization in GW mechanism, we rewrite the potential as

$$V_{\text{eff}} = k\epsilon v_0^2 + 4k e^{-4kL}(v_L - v_0 e^{-\epsilon kL})^2 \left(1 + \frac{\epsilon}{4}\right) - k\epsilon v_0 e^{-(4+\epsilon)kL}(2v_L - v_0 e^{-\epsilon kL}), \quad (46)$$

where $z_L = e^{kL}$ is utilized. This potential can be approximately minimized at

$$kr_c \equiv \frac{kL}{\pi} \simeq \left(\frac{4}{\pi}\right) \frac{k^2}{m_2} \ln \left[\frac{v_0}{v_L}\right]. \quad (47)$$

It is seen that the mechanism requires only

$$\frac{m^2}{k^2} \simeq \mathcal{O}(10), \quad (48)$$

in order to realize $kr_c \sim 10$, which is needed for solving the hierarchy problem in RS background. Therefore, it can be concluded that fine-tuning among parameters is not required to stabilize the configuration of radion which can play a crucial role in this approach for the gauge hierarchy problem. Finally, a numerical example is given by

$$\frac{v_0}{v_L} = 1.5, \quad \frac{m}{k} = 0.2, \quad kr_c = 12. \quad (49)$$

The effects of the radion on the oblique parameters, S , T , and U [29, 30, 31], have been evaluated by using an effective theory approach in [32]. As shown in [32], in the absence of a curvature-scalar Higgs mixing operator such as $\xi \mathcal{R} H^\dagger H$, the magnitude of the contribution to the oblique parameters from the radion can be small. On the other hand, in the presence of the mixing operator, the corrections become large due to the modified radion-Higgs couplings. As the results, the magnitude of fine-tuning among model parameters should be increased to achieve the Higgs mass larger than a few hundred GeV. Therefore, there are two options to protect the oblique parameters within an experimentally allowed region. One is to assume the absence of the curvature-scalar Higgs mixing operator. The other is to tune the parameters ξ and $v_{\text{EW}}/(M_{\text{pl}} e^{-kr_c})$ so that the oblique parameter are within an allowed region (see the Fig. 5 in [32] for an allowed region of ξ and $v_{\text{EW}}/(M_{\text{pl}} e^{-kr_c})$ given by a numerical calculation). The above options to control effects from radion can be generically taken for actual models discussed in the following subsections.

At the end of this subsection, it is worth commenting on realizations of the mechanism itself under various BCs discussed in the previous section. Since the Neumann BC including the effect from brane potential with huge boundary quartic coupling, which is imposed in the GW mechanism, is equivalent to the Dirichlet BC, the GW mechanism can work in the (D, D) BCs case with appropriate VEVs. Moreover, the mechanism can be also realized in (D, N) and (N, D) BCs including the effect from brane potential with large boundary quartic coupling while it cannot work in the absence of brane potentials. These are summarized in Tab 1. We also analyze the VEV profile when brane localized potentials are introduced. The Fig. 1 shows the VEV profile with the boundary quartic coupling. The drastical change of VEV profile in the (D, N) case can be seen when imposing larger boundary coupling, $\tilde{\lambda}_L \gtrsim \mathcal{O}(1)$. This significant change compared to the (N, D) case, $\tilde{\lambda}_0 \gtrsim \mathcal{O}(0.1)$ is explained as follows: The VEV profiles of both (D, D) and (N, D) cases in the absence of brane potential sharply localized on the IR brane while the profile of (D, N) case gently localized on the UV brane. potential), The required potential energy at the UV brane which changes the VEV profile of (N, D) case is smaller than one at IR boundary. It should drastically change the profile of (D, N) case. In other word, the form of VEV profile in the (N, D) case is more sensitive to the effect of brane potential than (D, N) or (D, D) cases. As for (N, N) case with large boundary potential, it is just the

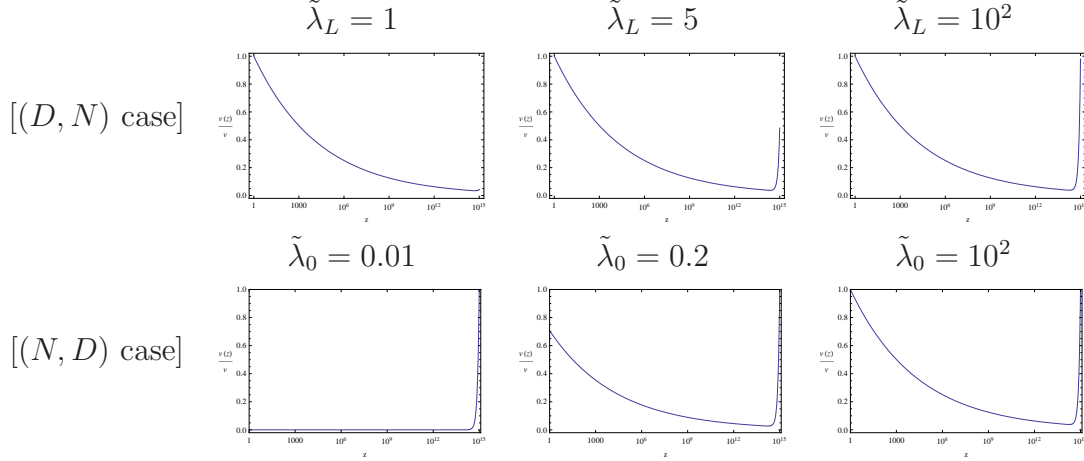


Figure 1: Dependence of VEV profile on the boundary coupling. We take the same values for other parameters as ones in Tab. 1.

case of original GW mechanism, and it has been shown to work well [26].

3.2 GW mechanism with bulk SM Higgs

In this subsection, we study a possibility of bulk SM Higgs with brane fermions, which is one of simple extensions of the SM. The constraints on the KK scale, $m_{KK} = \pi/L$ in flat five-dimensional spacetime from the EW precision measurements have been discussed in refs. [33, 34, 35, 36, 37]: $m_{KK} > 1.7$ TeV (90% CL) from the experimentally observed value of the Fermi constant [33], $m_{KK} \gtrsim 2.5$ TeV from the leptonic Z width [34], $m_{KK} \gtrsim 3.8$ TeV (95% CL) from a global fit of measurements of the Fermi constant, $\Gamma(Z \rightarrow f\bar{f})$, atomic parity violation, Weinberg angle, and W boson mass etc. when $m_H < 260$ GeV [35], $m_{KK} > 3.5$ TeV from a global fit of Fermi constant, Z , W , top masses, Z widths, asymmetries in Z decays⁸ [36], $m_{KK} > 85$ TeV from $K-\bar{K}$ and $D-\bar{D}$ mixing in bulk generation scenario where the first two generations live in the bulk together with the gauge multiplets and one of two Higgs fields [37], $m_{KK} > 1.52$ TeV (95% CL) from the measurement of the Fermi constant in $SU(2)$ -brane scenario [38, 39] from the Z leptonic width [37]. On the other hand, there is a phenomenologically interesting predictions in addition to the presence of KK particles in a higher-dimensional model, which is the *Yukawa deviation* [7, 21]. The Yukawa deviation is a phenomenon that the Yukawa coupling is smaller than the naive SM expectation, i.e. the SM fermion mass divided by the Higgs VEV. Such deviation can generally occur in multi-Higgs models, e.g. minimal supersymmetric standard model (MSSM). However, it has been pointed out that the Yukawa deviation can be induced from the presence of brane localized Higgs potential, which leads to deformed wave-function profile in the bulk for zero-

⁸ $m_{KK} > 4.3$ TeV if the Higgs is confined to brane [36].

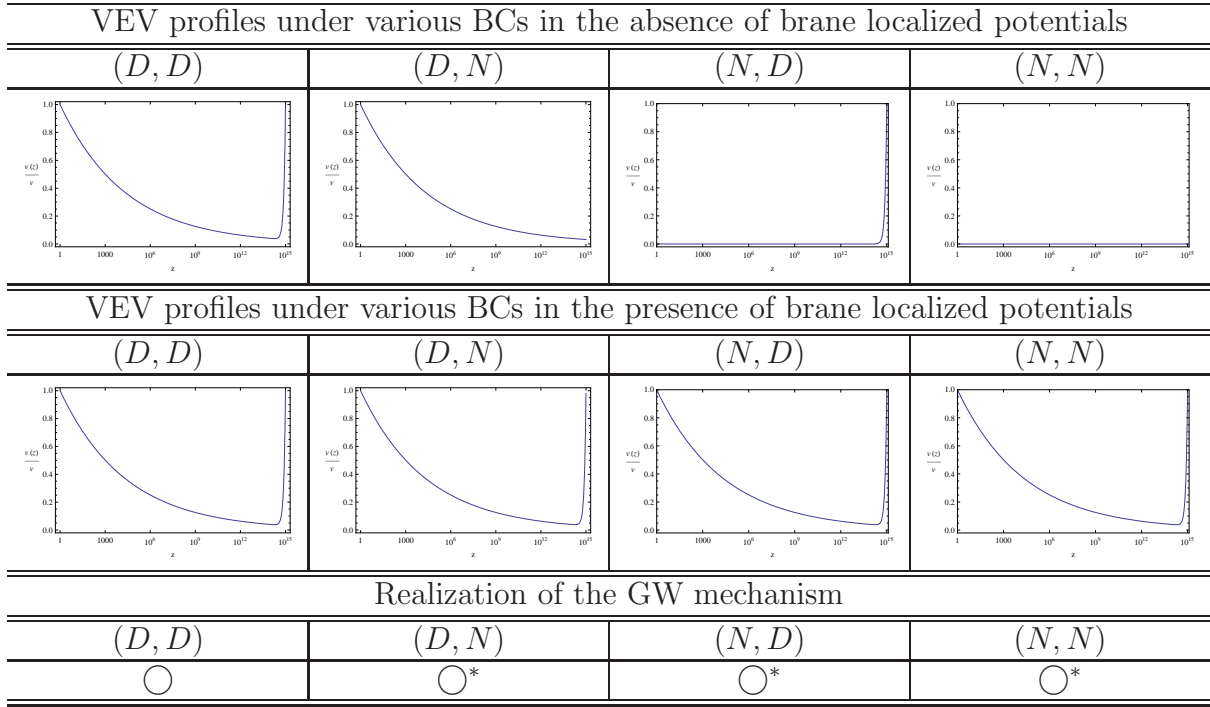


Table 1: VEV profiles and realization of GW mechanism: We redefine all dimension-full parameters as $v \equiv v_{0,1,2,L} \equiv \tilde{v}_{0,1,2,L} M_{\text{pl}}^{3/2}$, $\lambda_{0,L} \equiv \tilde{\lambda}_{0,L} M_{\text{pl}}^{-2}$ and $k \equiv \tilde{k} M_{\text{pl}}$, and take dimensionless parameters as $\tilde{v}_{0,1,2,L} = \tilde{k} = 1$ and $\nu = 2.1$, and the Planck scale as $M_{\text{pl}} = 2.4 \times 10^{18}$ GeV. Notice that the bulk scalar is canonically normalized at the UV brane and hence the values of VEV at the IR brane is one for unnormalized scalar field, which should be canonically normalized for the four-dimensional effective theory later. In the upper figures, the rapid changes of profiles near the boundary $z = z_L$ in the (D, D) and (N, D) cases are due to the Dirichlet BCs at $z = z_L$. In the (D, N) case of upper figure, the VEV at IR brane becomes smaller than Planck scale but is still finite, $v(z_L)/v \sim 0.03$. In the lower figures, we take the brane localized potential as given in (67) with large boundary couplings, $\tilde{\lambda}_{0,L} = 10^2$, for the (D, N) and (N, D) cases, and utilize the approximated solution of EOM at huge boundary coupling limit, $\tilde{\lambda}_{0,L} \gg 1$, for the (N, N) case which is presented in the GW mechanism [26]. It can be seen that the Neumann type BC with brane localized potential including a huge quartic coupling becomes equivalent to the non-vanishing Dirichlet BC. ○* means that the GW mechanism can work in the case that Neumann BC includes effects of brane localized potentials with huge boundary quartic coupling. The figure for (D, D) case without the brane localized potentials is the same as one for (D, D) case with brane potentials.

mode physical Higgs, in extra-dimensional setup even if there exists only one Higgs doublet [21]. Furthermore, the Dirichlet Higgs model where extra-dimensional BCs are Dirichlet type predicts the maximal Yukawa deviation with brane localized SM fermions [7]. How about a reliability of the Yukawa deviation in warped extra-dimension? We have shown the VEV profile of a bulk scalar field in all cases of BCs, (D, D) , (D, N) , (N, D) , and (N, N) . As shown in the previous section, the VEV profile localizes toward to the UV brane due to the Dirichlet BC in the (D, N) case. Such kind of model will be generically problematic when the SM gauge fields are in the bulk and the zero mode of bulk scalar field is identified with the SM Higgs boson. Since the gauge boson masses can be obtained from⁹

$$S = - \int d^4x \int_0^L dy e^{-2\sigma} \left[\frac{e^2}{4s_W} v(y)^2 W_\mu^+(x, y) W^{-\mu}(x, y) + \frac{e^2}{2(\sin 2\theta_W)^2} v(y)^2 Z_\mu(x, y) Z^\mu(x, y) \right], \quad (50)$$

bulk masses depend on the VEV profile. If the profile localizes toward to the UV brane, the realistic values of gauge boson masses cannot be reproduced at the IR brane. For the (D, D) and (N, D) BC cases, the VEV profiles localize toward to the IR brane. Therefore, SM gauge boson masses would be realized to be the same as the SM for W and Z bosons if effects from the bulk mass could be enough small. However, there are not solutions of the KK equation for the physical Higgs (quantum) field because of the presence of warp factor and Dirichlet BC at IR brane unlike the case of flat extra-dimension [7, 23]. Finally, (N, N) case cannot lead to non-trivial solutions of EOM (VEV). We can now conclude that the Yukawa deviation cannot occur in a realistic warped extra-dimensional model with brane fermions, bulk Higgs and gauge bosons even when there exists the brane-localized Higgs potential unlike a flat extra-dimension model [7, 21, 23].

Next, we discuss models with the bulk SM Higgs and fermions. One of important models with the bulk SM field in a flat five-dimensional spacetime is the UED [18]. In the work, it has been pointed out that the bound on the size of extra-dimension is weakened compared with other models including brane SM fermions and generations etc. due to the presence of KK parity that is consistent with an assumption that all SM fields live in the bulk. Furthermore, such kind of parity can also make the lightest KK particle a candidate for DM. The lower bound on the KK scale from the EW precision measurements is $m_{KK} \gtrsim 250$ GeV in the originally proposed UED model when the Higgs mass is relatively heavy as $m_H \simeq 950$ GeV (90% CL) [18, 41]. More general setup of the UED models have been discussed in [21, 22, 42]. The ref. [21] has also studied bulk fermion scenario in a case that the brane-localized potentials are introduced. Such kind of setup can also lead to the Yukawa deviation as discussed above. However, in order to realize a detectable size

⁹A detailed deviation is given in the Appendix B.

of the deviation at the LHC experiment, the relatively strong coupling is needed in the model with bulk SM fermions. The work [22] gave the complete computations of the KK expansion of the Higgs and gauge bosons in the UED model with brane localized potential by treating the potential as a small perturbation, and checked that the ρ parameter is not altered by effects from the potential. An alternatively generalized model from the UED [42] have analyzed effects from the existence of the brane localized kinetic and mass terms upon the extra-dimensional wave-functions profiles. A different approach to break EW symmetry from the UED is the Dirichlet Higgs model [7, 23, 24]. The model has a different structure of the Higgs sector from that of the UED, that is, the gauge symmetry is broken by non-zero Dirichlet BCs on the bulk Higgs field, and there are not any quartic interactions. As the results of this setup, the zero mode of the Higgs disappears and its lowest (first) KK mode couples to the zero modes of other SM fields with a suppression factor $2\sqrt{2}/\pi \simeq 0.9$. The detailed phenomenological aspects of this model for the LHC experiments have been discussed in [23]. The most important prediction of this model is that the physical Higgs mass is equal to the KK scale, $m_H = m_{KK}$, and the current EW precision measurements limit the mass to $430 \text{ GeV} \lesssim m_H \lesssim 500 \text{ GeV}$. The ref. [24] has pointed out that the $\mathcal{O}(\sqrt{s})$ growth of the longitudinal W^+W^- elastic scattering amplitude is exactly cancelled and hence can be unitarized by exchange of infinite towers of KK Higgs and resultant amplitude scales linearly with the scattering energy $\propto \sqrt{s}$. It has been also found that a tree level partial wave unitarity condition is satisfied up to 6.7(5.7) TeV for the KK scale $m_{KK} = 430(500) \text{ GeV}$. As mentioned above, simple extensions of the UED, which is constructed on the flat five dimensional spacetime, leads to deviations of the Higgs coupling. The LHC experiment would check such predictions. How about in a warped case with bulk SM Higgs?

Regarding to the deviations of the Higgs coupling from the SM expectation, a $SO(5) \times U(1)$ GHU model [19, 20] lead to suppressed coupling for WWH , ZZH , and Yukawa interactions by a factor $\cos\theta_H$ where θ_H is the Wilson line phase. Since $\theta_H = \pi/2$ can be dynamically realized in the model, the couplings for the above interactions vanish. As the results, the Higgs becomes stable, and thus, it can be a candidate for DM [43, 44]¹⁰. The mass of the Higgs in the model is predicted in a region $70 \text{ GeV} \leq m_H \leq 135 \text{ GeV}$ for the warp factor $10^5 \leq z_L \leq 10^{15}$.

The simplest model with the bulk Higgs on the warped extra-dimension to reproduce the SM on the four-dimensional brane is still the *bulk SM* [27]. In the work [27], two kind of scenarios have been proposed. One is that all SM particles live in the bulk and the other is that only Higgs is brane field while other SM particles are in the bulk. The work

¹⁰Another candidate for DM in $SO(5) \times U(1)_X$ GHU model can be realized by imposing an anti-periodic BC for a bulk field presented in [45].

has pointed out that the mass of the first KK excitation of the W boson should be larger than 9 TeV, whose constraint comes from the EW precision measurements¹¹. This bound is certainly weakened from that of [46]. However, the work [27] pointed out that the simple bulk SM where all SM fields are in the bulk should be discarded. The reason is as follows: In the model, the following potential of the five-dimensional Higgs field with a negative mass squared is taken,

$$\mathcal{V} = \frac{\lambda_5}{2} |\Phi|^4 - \mu^2 |\Phi|^2, \quad (51)$$

where λ_5 is a quartic coupling in five dimensions, and the VEV is assumed to develop as constant in the bulk. The VEV should generate the bulk mass term for the gauge boson, m_V . In order to reproduce the gauge boson masses and conserve kz_L^{-1} of $\mathcal{O}(\text{TeV})$, the magnitude of m_V should be around $\mathcal{O}(10^2 \text{ GeV})$. However, the natural value for m_V in the RS background would be the same order as k which is about M_{pl} . In other words, the gauge hierarchy is not solved at all in this simple bulk SM because the smallness of m_V requires small μ , which should be around the EW scale. Therefore it was concluded that the brane Higgs is the only choice to be able to avoid the above fine-tuning of the Higgs mass in the simple extension of the SM to RS background¹².

Next, let us consider an application of the above consideration to the GW mechanism with the bulk SM Higgs. We start with the following question: *Can the bulk SM Higgs stabilize the size of the warped extra-dimension?* (Here we neglect the fine-tuning problem of the bulk Higgs mass.) The answer for the question is *No*. The reason is as follows. The most essential term in the potential of GW mechanism is the second one in (46), which makes the mechanism work. Appropriate sizes of v_L and $v_0 e^{-\epsilon k L}$ makes the global minimum in the potential. Then the simple numerical game can give a viable example to realize the scenario without extreme fine-tuning among parameters as in (49). The example seems natural for choice of the magnitude of parameters, that is, all parameters are of the order of the Planck scale (in the basis where graviton is canonically normalized at the Planck brane). This means that since the GW mechanism corresponds to (D, D) type BCs, the dynamical scale of the Higgs field on the IR brane is just determined by the scale $v_2 = v_L$, which is the Planck scale. In other words, the physical scale of the Higgs on IR brane such as its mass depends only on the fundamental scale. Similar situation has been discussed in the Dirichlet Higgs model [7, 23, 24] for a five dimensional flat metric. In the model, Higgs mass becomes the compactification scale and the EW symmetry breaking occurs at the scale $v_1 = v_2 = v_L$, which can be taken at the usual EW scale in the case. On the

¹¹In a case that the SM gauge bosons live in the bulk while leptons and quarks are on the brane [46, 47], the KK scalar should be larger than 23 TeV, which is obtained from the EW precision measurements of the leptonic width of Z , atomic parity violation, and deep inelastic neutrino scattering [46].

¹²Notice that the assumption of constant VEV profile in [27] is difficult to realize as shown in Tab. 1.

other hand, in this application of the Higgs as the GW scalar, the exact (D, D) should be taken in order to avoid a strong coupling for the boundary quartic interaction for Higgs. This means that there is no boundary Higgs potential unlike the original RS setup for the SM sector. In a simple bulk SM with the bulk Higgs, if the VEV profile sufficiently localizes to the IR brane, the gauge boson masses might be reproduced. Such situation can be realized in the (D, D) and (N, D) cases without introducing large boundary coupling of the Higgs at IR brane as shown in Tab. 1. However, there are non-negligible contributions to the T parameter in such kind of models. An introduction of additional symmetry $SU(2)_R \times U(1)_{B-L}$ is one of approaches to suppress contributions to the T parameter. We focus on the model in the next subsection, and discuss the GW in the model.

3.3 GW mechanism in left-right model with custodial symmetry

In this section, we study a model with custodial symmetry in warped space. If we consider a model with brane Higgs and bulk gauge field, the KK states of gauge bosons contribute to the EW observables, which can be understood in terms of S , T , and U parameters [29, 30, 31]. Such a model is severely constrained by the EW precision measurements. The introduction of an additional symmetry, $SU(2)_R \times U(1)_{B-L}$, has been proposed to avoid such constraints, especially for the T parameter [28].

The model [28] starts with the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry in the bulk. The additional $SU(2)_R$ symmetry is broken to $U(1)_R$ by BCs on gauge fields at the UV brane to reproduce the usual EW symmetry while conserving the $SU(2)_R$ at the IR brane. Then the remaining $U(1)_R \times U(1)_{B-L}$ is spontaneously broken down to $U(1)_Y$ at the UV brane. The bulk action for the gauge and fermion sectors is given by

$$S = \int d^5x \sqrt{-G} (\mathcal{L}_g + \mathcal{L}_f), \quad (52)$$

where \mathcal{L}_g and \mathcal{L}_f are the Lagrangian for the gauge and fermions sectors given as

$$\begin{aligned} \mathcal{L}_g = & -\frac{1}{4} \sqrt{-G} (\text{tr} W_{MN} W^{MN} + \text{tr} \tilde{W}_{MN} \tilde{W}^{MN} + \text{tr} \tilde{B}_{MN} \tilde{B}^{MN} + \text{tr} F_{MN} F^{MN} \\ & + G^{MN} (D_M \Sigma)^\dagger (D_N \Sigma) + \mathcal{V}(\Sigma)), \end{aligned} \quad (53)$$

$$\mathcal{L}_f = \sqrt{-G} (i \bar{\Psi} \Gamma^M D_M \Psi - \epsilon(y) c_\Psi \bar{\Psi} \Psi). \quad (54)$$

The W_{MN} , \tilde{W}_{MN} , \tilde{B}_{MN} , and F_{MN} are the field strength for $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$, and $SU(3)_C$ of gauge groups, respectively, and Σ is a $SU(2)_R$ triplet Higgs, which breaks $SU(2)_R$ to $U(1)_R$ in the bulk at scale below k . The $\epsilon(y)$ is the sign function and c_Ψ is a parameter which determines the localization of the zero mode so that the wave-function profile localizes towards to the UV (IR) brane for $c_\Psi > 1/2$ ($< 1/2$) [48, 49]. After the

NG mode of Σ is eaten by the $SU(2)_R$ gauge field, the action for the gauge sector can be rewritten by

$$\mathcal{L}_g = -\frac{1}{4}\sqrt{-G}(\text{tr}W_{MN}W^{MN} + \text{tr}\tilde{W}_{MN}\tilde{W}^{MN} + \text{tr}\tilde{B}_{MN}\tilde{B}^{MN} + \text{tr}F_{MN}F^{MN} + \tilde{M}^2|\tilde{W}^\pm|^2). \quad (55)$$

The vanishing \tilde{M} corresponds to the unbroken $SU(2)_R$ in the bulk. The UV brane includes fields to break $U(1)_R \times U(1)_{B-L}$ to $U(1)_Y$ and the IR brane contains the usual SM Higgs field, which is described by a bidoublet under $SU(2)_L \times SU(2)_R$. The assignment for the gauge fields under $S^1/Z_2 \times Z'_2$ orbifold are given by (UV, IR) = $(-, +)$ for $\tilde{W}_\mu^{1,2}$ and $(+, +)$ for other gauge fields. The $U(1)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ breaking occurs through a VEV at the UV brane. The gauge fields related to the additional $SU(2)_R$ can be written as

$$Z'_\mu \equiv \frac{\tilde{g}_5 \tilde{W}_\mu^3 - \tilde{g}'_5 \tilde{B}_\mu}{\sqrt{\tilde{g}_5^2 + \tilde{g}'_5{}^2}} \quad \text{and} \quad B_\mu \equiv \frac{\tilde{g}'_5 \tilde{W}_\mu^3 - \tilde{g}_5 \tilde{B}_\mu}{\sqrt{\tilde{g}_5^2 + \tilde{g}'_5{}^2}}, \quad (56)$$

where the covariant derivative is defined by $D_M \equiv \partial_M - i(g_5 W_M^a \tau_{aL} + \tilde{g}_5 \tilde{M}_M^a \tau_{aR} + \tilde{g}'_5 \tilde{B}_M \tilde{Y})$ with $\tilde{Y} = (B - L)/2$. The hypercharge coupling, Z'_5 coupling, and \tilde{B} - \tilde{W}^3 mixing angle are defined by

$$g'_5 = \frac{\tilde{g}'_5 \tilde{g}_5}{\sqrt{\tilde{g}_5^2 + \tilde{g}'_5{}^2}}, \quad g_{Z'5} = \tilde{g}_5^2 + \tilde{g}'_5{}^2, \quad \sin \theta'_W = \frac{\tilde{g}'_5}{g_{Z'5}}. \quad (57)$$

where we can write four dimensional gauge couplings as $g = g_5/\sqrt{L}$, $g' = g'_5/\sqrt{L}$, $\tilde{g} = \tilde{g}_5/\sqrt{L}$, $\tilde{g}' = \tilde{g}'_5/\sqrt{L}$, and $g_{Z'} = g_{Z'5}/\sqrt{L}$. For the fermion sector, the usual right-handed fermions should be promoted to doublets under $SU(2)_R$ because the fermions are bulk fields and there is $SU(2)_R$ symmetry on the bulk.

Under the above setup, the EW fit has been discussed in terms of four-dimensional effective Lagrangian after integrating out the heavy modes [50, 51, 52, 53, 54]. The dimension-six operators,

$$\begin{aligned} \mathcal{L}_6 = & \frac{1}{16\pi^2 v^2} [gg'sH^\dagger \tau^a H B^{\mu\nu} W_{a\mu\nu} + (-t)((D^\mu H)^\dagger H)(H^\dagger D_\mu H) \\ & + (-ix)\bar{\psi}\gamma^\mu \tau^a \psi (D_\mu H)^\dagger \tau_a H + (-iy)\bar{\psi}\gamma^\mu \psi (D_\mu H)^\dagger H + V\bar{\psi}\psi\bar{\psi}\psi + h.c.], \end{aligned} \quad (58)$$

are important for the fit, where x , y , and V generally take as different values (couplings) for each fermion. The first and second terms are higher-dimensional operators corresponding to the gauge kinetic and mass terms for gauge fields, respectively, and the terms in the second line (58) correspond to the fermion sector. When the higher dimensional operators can be written as

$$x = ag^2 \quad \text{and} \quad y = ag'^2 Y Y_H, \quad (59)$$

these effects can be translated into oblique parameter by performing the following field redefinition,

$$W_3 \rightarrow W_3 \left(1 - g^2 \frac{a}{64\pi^2}\right) + Bgg' \frac{a}{64\pi^2}, \quad W^\pm \rightarrow W^\pm \left(1 - g^2 \frac{a}{64\pi^2}\right), \quad (60)$$

$$B \rightarrow B \left(1 - g'^2 \frac{a}{64\pi^2}\right) + W_3gg' \frac{a}{64\pi^2}, \quad (61)$$

where Y and Y_H are hypercharges of each fermion and Higgs, respectively. Then, the parameters s and t , and this redefinition give

$$S = \frac{s}{2\pi} + \frac{a}{2\pi}, \quad \text{and} \quad T = \frac{t}{8\pi e^2} + \frac{ag'^2}{8\pi e^2}. \quad (62)$$

Finally, the S and T parameters in this model can be estimated as [28]

$$S \simeq 2\pi \left(\frac{v_{\text{EW}}}{k} z_L\right)^2 \quad \text{and} \quad T \simeq \frac{\pi \tilde{g}^2}{2 e^2} \left(\frac{v_{\text{EW}}}{k} z_L\right)^2 kL \frac{\tilde{M}^2}{4k^2}. \quad (63)$$

Notice that the main contribution to T is proportional to the bulk mass of $SU(2)_R$ gauge boson. Therefore, if the bulk $SU(2)_R$ is unbroken, the main contribution becomes the next leading order of $\mathcal{O}((kL)^0)$.

The above model with the bulk custodial symmetry is one of fascinating models to be extended from the SM to five-dimensional model on warped space. This can pass severe EW precision tests due to the custodial protection and have some predictions for collider signatures because of a lower bound of the KK scale around a few TeV. Our next task is to consider a simplification of warped extra-dimensional models. Towards this purpose we start with the question: *Can the triplet Higgs under $SU(2)_R$ stabilize the radius of extra dimension?* The answer for the question is *Yes*. The essential points for the GW mechanism are (49), namely the VEVs around the five-dimensional fundamental scale of order the Planck one and slightly smaller bulk mass of the triplet Higgs than the AdS curvature scale. On the other hand, in order to suppress the contribution from the KK sector to the T parameter in the framework of the model with the custodial symmetry, unbroken $SU(2)_R$ in the bulk is favored. Of course, broken $SU(2)_R$ with $\tilde{M} \ll k$ is also possible as mentioned above. The point for working GW mechanism and that of custodial protection can be completely compatible with each other. Therefore, we can achieve a *radius stabilization by the $SU(2)_R$ triplet Higgs*. In this scenario, the $SU(2)_R$ is broken at both boundaries by appropriate BCs which should be either Neumann BCs with large quartic Higgs coupling at boundaries or (D, D) BCs to stabilize the radius as in the original GW mechanism. And we can always take the bulk potential $\mathcal{V}(\Sigma)$ of the triplet Higgs as favored for the custodial protection. In this direction, an additional bulk scalar is not needed. Therefore, we conclude that this scenario is one of the simplest extensions of the

SM to five-dimensional model on the warped space to realize radius stabilization and easily pass the EW precision tests without any fine-tunings.

We have shown that the GW radius stabilization can be achieved by the $SU(2)_R$ triplet Higgs in the (D, D) case (or corresponding replacement to Neumann BC with large boundary coupling) as one of the simplest extensions of the SM to five-dimensional model on the warped space. The introduction of the triplet Higgs is one of options discussed in the model [28]. The sole role of the field is to spontaneously break $SU(2)_R$ to $U(1)_R$ at a mass scale below curvature scale. However, it is not necessary for the protection of T parameter to introduce the field, rather, an option without the triplet (unbroken $SU(2)_R$) is more favored for the protection. Therefore, the model without the triplet is still simple where the radius stabilization is realized by the conventional GW mechanism (with gauge singlet bulk scalar). Finally, we comment on other possibility of extension of the SM. In a left-right model with custodial symmetry [28], the SM Higgs exactly localizes at the IR brane. It might be still possible that this Higgs becomes bulk field if the wave function sufficiently localizes towards the IR brane so that the gauge boson masses are reproduced. When the (D, D) or (N, D) with large boundary coupling at UV brane are taken, the bulk Higgs could stabilize the radius of extra-dimension. The boundary quartic coupling at IR brane should not be large because the Higgs corresponds to the SM one in this case. In such models, one would have to discuss generating Yukawa hierarchies and flavor changing neutral currents because an overlap among the Higgs and fermions near the UV brane are not suppressed. Such considerations would be worth studying further.

4 Summary

We have studied implications of generalized non-zero Dirichlet BC along with the ordinary Neumann one on a bulk scalar in the RS warped compactification. First we have shown profiles of VEV of the scalar under the general BCs. These BCs are described by combinations of the Neumann and Dirichlet types as, $(UV, IR) = (D, D), (D, N), (N, D)$, and (N, N) . It has been clarified that the VEV profile localizes toward to the IR brane in the (D, D) and (N, D) BCs while it localizes toward to the UV brane in the (D, N) case. And we have also shown that there is not a non-zero solution of EOM in the (N, N) case.

We have also investigated GW mechanism in several setups with the general boundary conditions of the bulk scalar field. We have shown that the GW mechanism can work under non-zero Dirichlet BCs with appropriate size of VEVs. (i) First we have considered the application: the bulk SM Higgs as a bulk scalar for the GW mechanism. We have also reviewed related topics: In this application, the (D, D) BCs should be taken in order to avoid a strong coupling for the boundary quartic interaction for the Higgs while realizing

the GW mechanism. Furthermore, the bulk SM where all SM fields live in the bulk cannot still solve the hierarchy problem because of the required bulk mass of the order of the EW scale to reproduce the SM gauge boson masses. This difficulty cannot be avoided under all combinations of BCs formulated in the above even if the brane localized Higgs potential are introduced. Therefore, we conclude that the bulk SM Higgs cannot be a GW stabilizer unless we allow unnaturally small bulk mass compared to the fundamental scale. (ii) We have also discussed an application of the triplet Higgs under additional $SU(2)_R$ symmetry to a bulk scalar in the GW mechanism. In this scenario, all the requirements to realize the GW mechanism and custodial protection for T parameter in $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model are completely compatible with each other. Therefore, we conclude that the radius can be stabilized by the $SU(2)_R$ triplet Higgs.

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A Case with brane localized potentials

A.1 Action and BCs

In this appendix, we give formulation and VEV profile in a case with brane localized scalar potentials. We should start with the following action of the bulk scalar in behalf of (5),

$$S = \int d^4x \int_0^L dy e^{-4\sigma} [-e^{2\sigma} |\partial_\mu \Phi|^2 - |\partial_y \Phi|^2 - \mathcal{V} - \delta(y)V_0 - \delta(y-L)V_L]. \quad (64)$$

The variation of the action is given by

$$\begin{aligned} \delta S = & \int d^4x \int_0^L dy e^{-4\sigma} \left[\delta\Phi_X \left(\mathcal{P}\Phi_X - \frac{\partial\mathcal{V}}{\partial\Phi_X} \right) \right. \\ & \left. + \delta(y)\delta\Phi_X \left(+\partial_y\Phi_X - \frac{\partial V_0}{\partial\Phi_X} \right) + \delta(y-L)\delta\Phi_X \left(-\partial_y\Phi_X - \frac{\partial V_L}{\partial\Phi_X} \right) \right], \end{aligned} \quad (65)$$

and thus the Neumann BC should be modified from (11) to

$$\pm \partial_y \Phi_X - \frac{\partial V_\eta}{\partial \Phi_X} \Big|_{y=\eta} = 0, \quad (66)$$

while the Dirichlet one is the same as (10) even if the brane localized potentials are introduced. In this Appendix, we take the brane localized potentials as

$$V_\eta = \lambda_\eta \left(|\Phi|^2 - \frac{v_\eta^2}{2} \right)^2 = \frac{\lambda_\eta}{4} (\Phi_R^2 + \Phi_I^2 - v_\eta^2)^2. \quad (67)$$

The free action for the physical Higgs and NG can be written as

$$\begin{aligned}
S = & \int d^4x \int_0^L dy e^{-4\sigma} \left[-\mathcal{V} - \frac{e^{2\sigma}}{2} (\partial_\mu \Phi_R + \partial_\mu \phi)^2 - \frac{e^{2\sigma}}{2} (\partial_\mu \Phi_I + \partial_\mu \chi)^2 \right. \\
& - \frac{1}{2} (\partial_y \Phi_R + \partial_y \phi)^2 - \frac{1}{2} (\partial_y \Phi_I + \partial_y \chi)^2 - \frac{\partial \mathcal{V}^c}{\partial \Phi_R} \phi - \frac{\partial \mathcal{V}^c}{\partial \Phi_I} \chi - \frac{1}{2} \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_R^2} \phi^2 - \frac{1}{2} \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_I^2} \chi^2 \\
& - \delta(y) \left(V_0 + \frac{\partial V_0^c}{\partial \Phi_R} \phi + \frac{\partial V_0^c}{\partial \Phi_I} \chi + \frac{1}{2} \frac{\partial^2 V_0^c}{\partial \Phi_R^2} \phi^2 + \frac{1}{2} \frac{\partial^2 V_0^c}{\partial \Phi_I^2} \chi^2 \right) \\
& \left. - \delta(y-L) \left(V_L + \frac{\partial V_L^c}{\partial \Phi_R} \phi + \frac{\partial V_L^c}{\partial \Phi_I} \chi + \frac{1}{2} \frac{\partial^2 V_L^c}{\partial \Phi_R^2} \phi^2 + \frac{1}{2} \frac{\partial^2 V_L^c}{\partial \Phi_I^2} \chi^2 \right) \right]. \quad (68)
\end{aligned}$$

The partial integrals for $-e^{-2\sigma}(\partial_\mu \Phi_R)(\partial_\mu \phi)$, $-e^{-2\sigma}(\partial_\mu \Phi_I)(\partial_\mu \chi)$, $-e^{-4\sigma}(\partial_y \Phi_R)(\partial_y \phi)$ and $-e^{-4\sigma}(\partial_y \Phi_I)(\partial_y \chi)$ in the integrand give

$$\begin{aligned}
S = & \int d^4x \int_0^L dy e^{-4\sigma} \left[-\mathcal{V} - \frac{e^{2\sigma}}{2} \{(\partial_\mu \Phi_R)^2 - 2\phi \square \Phi_R + (\partial_\mu \phi)^2\} \right. \\
& - \frac{e^{2\sigma}}{2} \{(\partial_\mu \Phi_I)^2 - 2\chi \square \Phi_I + (\partial_\mu \chi)^2\} - \frac{1}{2} \{(\partial_y \Phi_R)^2 - 2e^{4\sigma} \phi \partial_y (e^{-4\sigma} (\partial_y \Phi_R)) + (\partial_y \phi)^2\} \\
& - \frac{1}{2} \{(\partial_y \Phi_I)^2 - 2e^{4\sigma} \chi \partial_y (e^{-4\sigma} (\partial_y \Phi_I)) + (\partial_y \chi)^2\} - \frac{\partial \mathcal{V}^c}{\partial \Phi_R} \phi - \frac{\partial \mathcal{V}^c}{\partial \Phi_I} \chi \\
& - \frac{1}{2} \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_R^2} \phi^2 - \frac{1}{2} \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_I^2} \chi^2 \\
& - \delta(y) \left(V_0 - \phi \partial_y \Phi_R - \chi \partial_y \Phi_I + \frac{\partial V_0^c}{\partial \Phi_R} \phi + \frac{\partial V_0^c}{\partial \Phi_I} \chi + \frac{1}{2} \frac{\partial^2 V_0^c}{\partial \Phi_R^2} \phi^2 + \frac{1}{2} \frac{\partial^2 V_0^c}{\partial \Phi_I^2} \chi^2 \right) \\
& \left. - \delta(y-L) \left(V_L + \phi \partial_y \Phi_R + \chi \partial_y \Phi_I + \frac{\partial V_L^c}{\partial \Phi_R} \phi + \frac{\partial V_L^c}{\partial \Phi_I} \chi + \frac{1}{2} \frac{\partial^2 V_L^c}{\partial \Phi_R^2} \phi^2 + \frac{1}{2} \frac{\partial^2 V_L^c}{\partial \Phi_I^2} \chi^2 \right) \right]. \quad (69)
\end{aligned}$$

Here, note that the terms depending only on Φ_X , V_η , and \mathcal{V} are vanishing due to the EOM

and Neumann BCs. Therefore, we can obtain the following action,

$$\begin{aligned}
S = & \int d^4x \int_0^L dy e^{-4\sigma} \left[-\frac{e^{2\sigma}}{2} \{-2\phi \square \Phi_R + (\partial_\mu \phi)^2\} - \frac{e^{2\sigma}}{2} \{-2\chi \square \Phi_I + (\partial_\mu \chi)^2\} \right. \\
& -\frac{1}{2} \{-2e^{4\sigma} \phi \partial_y (e^{-4\sigma} (\partial_y \Phi_R)) + (\partial_y \phi)^2\} - \frac{1}{2} \{-2e^{4\sigma} \chi \partial_y (e^{-4\sigma} (\partial_y \Phi_I)) + (\partial_y \chi)^2\} \\
& -\frac{\partial \mathcal{V}^c}{\partial \Phi_R} \phi - \frac{\partial \mathcal{V}^c}{\partial \Phi_I} \chi - \frac{1}{2} \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_R^2} \phi^2 - \frac{1}{2} \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_I^2} \chi^2 \\
& -\delta(y) \left(-\phi \partial_y \Phi_R - \chi \partial_y \Phi_I + \frac{\partial V_0^c}{\partial \Phi_R} \phi + \frac{\partial V_0^c}{\partial \Phi_I} \chi + \frac{1}{2} \frac{\partial^2 V_0^c}{\partial \Phi_R^2} \phi^2 + \frac{1}{2} \frac{\partial^2 V_0^c}{\partial \Phi_I^2} \chi^2 \right) \\
& \left. -\delta(y-L) \left(+\phi \partial_y \Phi_R + \chi \partial_y \Phi_I + \frac{\partial V_L^c}{\partial \Phi_R} \phi + \frac{\partial V_L^c}{\partial \Phi_I} \chi + \frac{1}{2} \frac{\partial^2 V_L^c}{\partial \Phi_R^2} \phi^2 + \frac{1}{2} \frac{\partial^2 V_L^c}{\partial \Phi_I^2} \chi^2 \right) \right]. \tag{70}
\end{aligned}$$

We also notice that the linear terms of ϕ and χ vanishes because of the EOM and Neumann BCs for the Φ_X fields as

$$\begin{aligned}
S = & \int d^4x \int_0^L dy e^{-4\sigma} \left[-\frac{e^{2\sigma}}{2} (\partial_\mu \phi)^2 - \frac{e^{2\sigma}}{2} (\partial_\mu \chi)^2 - \frac{1}{2} (\partial_y \phi)^2 - \frac{1}{2} (\partial_y \chi)^2 \right. \\
& -\frac{1}{2} \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_R^2} \phi^2 - \frac{1}{2} \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_I^2} \chi^2 \\
& \left. -\delta(y) \left(\frac{1}{2} \frac{\partial^2 V_0^c}{\partial \Phi_R^2} \phi^2 + \frac{1}{2} \frac{\partial^2 V_0^c}{\partial \Phi_I^2} \chi^2 \right) - \delta(y-L) \left(\frac{1}{2} \frac{\partial^2 V_L^c}{\partial \Phi_R^2} \phi^2 + \frac{1}{2} \frac{\partial^2 V_L^c}{\partial \Phi_I^2} \chi^2 \right) \right]. \tag{71}
\end{aligned}$$

The partial integrals for each kinetic term make the action

$$\begin{aligned}
S = & \int d^4x \int_0^L dy e^{-4\sigma} \left[\frac{1}{2} \phi \left(e^{2\sigma} \square + e^{4\sigma} \partial_y e^{-4\sigma} \partial_y - \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_R^2} \right) \phi \right. \\
& + \frac{1}{2} \chi \left(e^{2\sigma} \square + e^{4\sigma} \partial_y e^{-4\sigma} \partial_y - \frac{\partial^2 \mathcal{V}^c}{\partial \Phi_I^2} \right) \chi \\
& -\frac{\delta(y)}{2} \left(-\phi \partial_y \phi - \chi \partial_y \chi + \frac{\partial^2 V_0^c}{\partial \Phi_R^2} \phi^2 + \frac{\partial^2 V_0^c}{\partial \Phi_I^2} \chi^2 \right) \\
& \left. -\frac{\delta(y-L)}{2} \left(+\phi \partial_y \phi + \chi \partial_y \chi + \frac{\partial^2 V_L^c}{\partial \Phi_R^2} \phi^2 + \frac{\partial^2 V_L^c}{\partial \Phi_I^2} \chi^2 \right) \right]. \tag{72}
\end{aligned}$$

It is seen that the Neumann BCs are modified from (25) as

$$\left(\pm \partial_y - \frac{\partial^2 V_\eta^c}{\partial \Phi_R^2} \right) f_n(y) \Big|_{y=\eta} = 0. \tag{73}$$

A.2 VEV profiles

We discuss the VEV profiles for the (D, N) and (N, D) BCs in the case that the boundary quartic coupling is finite.

A.2.1 (D, N) case

The BCs are given by

$$v(1) = v_1, \quad \partial_z v(z)|_{z=z_L} + \frac{\partial V_L^c}{\partial \Phi} \Big|_{z=z_L} = 0. \quad (74)$$

They are written down as

$$A + B = v_1, \quad (75)$$

$$k[A(\nu + 2)z_L^{\nu+2} - B(\nu - 2)z_L^{-(\nu-2)}] + \lambda_L(Az_L^{\nu+2} + Bz_L^{-(\nu-2)})[(Az_L^{\nu+2} + Bz_L^{-(\nu-2)})^2 - v_L^2] = 0. \quad (76)$$

One must note that when the coupling in the boundary potential, λ_η , becomes infinite, the Neumann BCs turn to the Dirichlet ones, $v(y)|_{y=\eta} \rightarrow v_\eta$. Similar situation has been discussed in the GW mechanism [26].

When the boundary quartic coupling is finite, the numerical calculation indicates $A \ll B \simeq z_L^{\nu-2}v_L$ as a solution of (75) and (76). Then the VEV profile can be approximated by

$$v(z) \simeq \left(v_1 - z_L^{\nu-2} \sqrt{v_L^2 + \frac{k(\nu-2)z_L^{-(\nu-2)}}{\lambda_L}} \right) z^{\nu+2} + \sqrt{v_L^2 + \frac{k(\nu-2)z_L^{-(\nu-2)}}{\lambda_L}} \left(\frac{z}{z_L} \right)^{\nu-2}. \quad (77)$$

A typical behavior of VEV profile in a case that the boundary coupling is finite is shown in the left figure of Fig. 1.

A.2.2 (N, D) case

The BCs are

$$\partial_z v(z)|_{z=1} + \frac{\partial V_0^c}{\partial \Phi} \Big|_{z=1} = 0, \quad v(z_L) = v_2. \quad (78)$$

They are written down as

$$k[A(\nu + 2) - B(\nu - 2)] - \lambda_0(A + B)[(A + B)^2 - v_0^2] = 0, \quad (79)$$

$$Az_L^{\nu+2} + Bz_L^{-(\nu-2)} = v_2. \quad (80)$$

Numerical calculation indicates $A \sim B \sim v_2/z_L^{\nu+2}$ and the VEV profile is approximated by

$$v(z) \simeq \left[v_2 - \frac{(k + \lambda_0 v_0 v_2) v_0}{2k\nu - \lambda_0 v_0^2 (z_L^{2\nu} - 1)} \right] \left(\frac{z}{z_L} \right)^{\nu+2} + \frac{(k + \lambda_0 v_0 v_2) v_0}{2k\nu - \lambda_0 v_0^2 (z_L^{2\nu} - 1)} \left(\frac{z}{z_L} \right)^{-(\nu-2)}. \quad (81)$$

The typical VEV profile is shown in the left figure of Fig. 1.

B Gauge sector

In this Appendix, we write down interactions of SM gauge field with the bulk SM Higgs to show the dependence of the gauge boson mass on the VEV profile. Then a deconstruction method is also presented, which gives profile of gauge field.

B.1 Interactions of gauge field with the bulk SM Higgs

First, we write down interactions of gauge field with the bulk SM Higgs. If the Higgs sector are also on the bulk, the Higgs kinetic term is

$$S_{kin} = \int d^4x \int_0^L dy \sqrt{-G} [-G^{MN} (D_M \Phi)^\dagger (D_N \Phi)], \quad (82)$$

where

$$\Phi \equiv \left(\frac{\varphi^+(x, y)}{\frac{v(y) + H(x, y) + i\chi(x, y)}{\sqrt{2}}} \right), \quad (83)$$

$$D_M \equiv \partial_M + ig_5 W_M^a T^a + ig'_5 B_M Y \quad (84)$$

$$= \partial_M + i \frac{g_5}{2} \begin{pmatrix} W_M^3 & W_M^1 - iW_M^2 \\ W_M^1 + iW_M^2 & -W_M^3 \end{pmatrix} + i \frac{g'_5}{2} \begin{pmatrix} B_M & 0 \\ 0 & B_M \end{pmatrix} \quad (85)$$

$$= \partial_M + i \frac{g_5}{\sqrt{2}} \begin{pmatrix} 0 & W_M^+ \\ W_M^- & 0 \end{pmatrix} + \frac{i}{2} \begin{pmatrix} g_5 W_M^3 + g'_5 B_M & 0 \\ 0 & -g_5 W_M^3 + g'_5 B_M \end{pmatrix}, \quad (86)$$

and g_5 and g'_5 are the gauge couplings in five dimension. Here we can write the Weinberg angle and the gauge couplings as

$$\sin \theta_W \equiv s_W \equiv \frac{g'_5}{\sqrt{g_5^2 + g'^2_5}} = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad \cos \theta_W \equiv c_W \equiv \frac{g_5}{\sqrt{g_5^2 + g'^2_5}} = \frac{g}{\sqrt{g^2 + g'^2}}, \quad (87)$$

$$e \equiv \frac{g_5 g'_5}{\sqrt{g_5^2 + g'^2_5}} = \frac{gg'}{\sqrt{g^2 + g'^2}}, \quad g_5 \equiv g\sqrt{L}, \quad g'_5 \equiv g'\sqrt{L}. \quad (88)$$

Then the covariant derivative (86) can be rewritten by

$$D_M = \partial_M + i \frac{g_5}{\sqrt{2}} \begin{pmatrix} 0 & W_M^+ \\ W_M^- & 0 \end{pmatrix} + \frac{i}{2} \sqrt{g_5^2 + g'^2_5} \begin{pmatrix} c_W W_M^3 + s_W B_M & 0 \\ 0 & -c_W W_M^3 + s_W B_M \end{pmatrix}. \quad (89)$$

Further, after defining

$$\begin{pmatrix} Z_M \\ A_M \end{pmatrix} \equiv \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W_M^3 \\ B_M \end{pmatrix}, \quad (90)$$

The covariant derivative becomes

$$D_M = \partial_M + i \frac{g_5}{\sqrt{2}} \begin{pmatrix} 0 & W_M^+ \\ W_M^- & 0 \end{pmatrix} + i \begin{pmatrix} \frac{g_5^2 - g'^2_5}{2\sqrt{g_5^2 + g'^2_5}} Z_M + e A_W & 0 \\ 0 & -\frac{\sqrt{g_5^2 + g'^2_5}}{2} Z_M \end{pmatrix}. \quad (91)$$

Here, we give useful relations among the Weinberg angle and gauge couplings,

$$\frac{g_5^2 - g_5'^2}{2\sqrt{g_5^2 + g_5'^2}} = \frac{\sqrt{g_5^2 + g_5'^2}(c_W^2 - s_W^2)}{2} = \frac{c_W^2 - s_W^2}{2s_W c_W} e = \frac{1}{\tan 2\theta_W}, \quad (92)$$

$$\sqrt{g_5^2 + g_5'^2} = \frac{e}{c_W s_W}, \quad g_5 = \frac{e}{s_W}. \quad (93)$$

Finally, we obtain

$$D_M = \partial_M + \frac{i}{\sqrt{2}} \frac{e}{s_W} \begin{pmatrix} 0 & W_M^+ \\ W_M^- & 0 \end{pmatrix} + ie \begin{pmatrix} \frac{1}{\tan 2\theta_W} Z_M + A_W & 0 \\ 0 & -\frac{1}{\sin 2\theta_W} Z_M \end{pmatrix}, \quad (94)$$

and thus,

$$\begin{aligned} & (D_M \Phi)^\dagger (D_N \Phi) \\ &= \left(\partial_M \varphi^- - \frac{ie}{2s_W} W_M^- (v(y) + H - i\chi) - ie \left(\frac{1}{\tan 2\theta_W} Z_M + A_M \right) \varphi^- \right) \\ & \times \left(\partial_N \varphi^+ + \frac{ie}{2s_W} W_N^- (v(y) + H + i\chi) + ie \left(\frac{1}{\tan 2\theta_W} Z_N + A_N \right) \varphi^+ \right) \\ & + \frac{1}{2} \left(\partial_M v(y) + \partial_M H - i\partial_M \chi - \frac{ie}{s_W} W_M^+ \varphi^- + \frac{ie}{\sin 2\theta_W} Z_M (v(y) + H - i\chi) \right) \\ & \times \frac{1}{2} \left(\partial_N v(y) + \partial_N H + i\partial_N \chi + \frac{ie}{s_W} W_N^- \varphi^+ - \frac{ie}{\sin 2\theta_W} Z_N (v(y) + H + i\chi) \right). \quad (95) \end{aligned}$$

The quadratic, cubic, and quartic terms are written down by

$$\begin{aligned} & (D_M \Phi)^\dagger (D_N \Phi) \Big|_{\text{quadratic}} \\ &= (\partial_M \varphi^-)(\partial_N \varphi^+) + \frac{e}{2s_W} [iv(y)(\partial_M \varphi^-)W_N^+ - iv(y)(\partial_N \varphi^+)W_M^-] \\ & + \frac{e^2}{4s_W} v(y)^2 W_N^+ W_M^- + \frac{1}{2} (\partial_M v(y))(\partial_N v(y)) + \frac{1}{2} (\partial_N v(y))(\partial_M H) \\ & + \frac{1}{2} (\partial_M v(y))(\partial_N H) + \frac{1}{2} [(\partial_M H)(\partial_N H) + (\partial_M \chi)(\partial_N \chi)] \\ & + \frac{e}{s_W} [i(\partial_M v(y))W_N^- \varphi^+ - i(\partial_N v(y))W_M^+ \varphi^-] \\ & + \frac{e}{2 \sin 2\theta_W} [iZ_M \{(\partial_N v(y))v(y) + (\partial_N v(y))(H - i\chi) + (\partial_N H + i\partial_N \chi)v(y)\} \\ & \quad - iZ_N \{(\partial_M v(y))v(y) + (\partial_M v(y))(H + i\chi) + (\partial_M H - i\partial_M \chi)v(y)\}] \\ & + \frac{e^2}{2(\sin 2\theta_W)^2} v(y)^2 Z_M Z_N, \quad (96) \end{aligned}$$

$$\begin{aligned}
& (D_M \Phi)^\dagger (D_N \Phi) \Big|_{\text{cubic}} \\
&= \frac{e}{2s_W} \left[iW_N (\partial_M \varphi^-) (H + i\chi) - iW_M (\partial_N \varphi^+) (H - i\chi) \right] \\
&+ e \left[i(\partial_M \varphi^-) \varphi^+ \left(\frac{Z_N}{\tan 2\theta_W} + A_N \right) - i(\partial_N \varphi^+) \varphi^- \left(\frac{Z_M}{\tan 2\theta_W} + A_M \right) \right] \\
&+ \frac{e^2}{2s_W^2} v(y) W_N^+ W_M^- H \\
&+ \frac{e^2}{2s_W} \left[v(y) W_N^+ \left(\frac{Z_M}{\tan 2\theta_W} + A_M \right) \varphi^- + v(y) W_M^- \left(\frac{Z_N}{\tan 2\theta_W} + A_N \right) \varphi^+ \right] \\
&+ \frac{e}{2s_W} \left[i(\partial_M H - i\partial_M \chi) W_N^- \varphi^+ - i(\partial_N H + i\partial_N \chi) W_M^+ \varphi^- \right] \\
&+ \frac{e}{2 \sin 2\theta_W} \left[iZ_M (\partial_N H + i\partial_N \chi) (H - i\chi) - iZ_N (\partial_M H - i\partial_M \chi) (H + i\chi) \right] \\
&- \frac{e^2}{2s_W \sin 2\theta_W} \left[W_N^- \varphi^+ Z_N v(y) + W_M^+ \varphi^- Z_N v(y) \right] + \frac{e^2}{(\sin 2\theta_W)^2} v(y) H Z_M Z_N, \quad (97)
\end{aligned}$$

$$\begin{aligned}
& |(D_M \Phi)^\dagger (D_N \Phi)|_{\text{quartic}} \\
&= \frac{e^2}{4s_W^2} W_N^+ W_M^- (H^2 + \chi^2) + \frac{e^2}{2(\sin \theta_W)^2} Z_N Z_M (H^2 + \chi^2) \\
&+ \frac{e^2}{2s_W} \left[W_N^+ (H + i\chi) \left(\frac{Z_M}{\tan 2\theta_W} + A_M \right) \varphi^- + W_M^+ (H - i\chi) \left(\frac{Z_N}{\tan 2\theta_W} + A_N \right) \varphi^+ \right] \\
&+ e^2 \left(\frac{Z_M}{\tan 2\theta_W} + A_M \right) \left(\frac{Z_N}{\tan 2\theta_W} + A_N \right) \varphi^+ \varphi^- + \frac{e^2}{2s_W^2} W_N^+ W_M^- \varphi^+ \varphi^- \\
&- \frac{e^2}{2s_W \sin 2\theta_W} \left[W_N^- \varphi^+ Z_M (H - i\chi) + W_M^- \varphi^- Z_N (H + i\chi) \right], \quad (98)
\end{aligned}$$

where we take $W_y(x, y)^\pm = Z_y(x, y) = 0$. We find from (96) that the gauge boson masses in (50) is obtained.

B.2 Deconstruction

For ones who are interested in obtaining gauge field profile, which is not directly related with the main discussion of this paper, we show a deconstruction method in Abelian case with flat five-dimensional setup. An extension to a warped case is straightforward.

The gauge kinetic term in Abelian case can be written as

$$\int_0^L dy \mathcal{L} = -\frac{1}{4} \int_0^L dy (\partial_M A_N - \partial_N A_M)^2 \quad (99)$$

$$\begin{aligned}
&= -\frac{1}{4} \int_0^L dy \left[(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + (\partial_\mu A_y)^2 + (\partial_y A_\mu)^2 + A_y \partial_y \partial_\mu A^\mu \right] \\
&+ \frac{1}{4} [A_y \partial_\mu A^\mu]_0^L, \quad (100)
\end{aligned}$$

where we operated an partial integral in the second line. If we take the Lorentz gauge, $\partial_\mu A^\mu = 0$, as a gauge fixing, (100) turns to

$$\int_0^L dy \mathcal{L} = -\frac{1}{4} \int_0^L dy (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{4} \int_0^L dy [(\partial_\mu A_y)^2 + (\partial_y A_\mu)^2]. \quad (101)$$

After latticizing the five-dimensional coordinate as

$$y_n \equiv na, \quad A_{\mu,n} \equiv A_\mu(y_n), \quad A_{y,n} \equiv A_y(y_n), \quad (102)$$

the second term (101) becomes

$$-\frac{1}{4} \int_0^L dy [(\partial_\mu A_y)^2 + (\partial_y A_\mu)^2] = -\frac{1}{4} \sum_{n=0}^N \left[(\partial_\mu A_{y,n})^2 + \left(\frac{A_{\mu,(n+1)} - A_{\mu,n}}{a} \right)^2 \right], \quad (103)$$

where a is a size of lattice and a periodicity, $A_{\mu,N+1} = A_{\mu,0}$, is assumed.

It might be instructive to show here about the gauge transformation before proceeding discussion. Let us consider the transformation for a bi-fundamental field as

$$\Phi_{n,n+1} \rightarrow U_n \Phi_{n,n+1} U_{n+1}^{-1}. \quad (104)$$

And we write the covariant derivative as

$$D_\mu \Phi_{n,n+1} \equiv \partial_\mu \Phi_{n,n+1} + ig A_{\mu n} \Phi_{n,n+1} - ig \Phi_{n,n+1} A_{\mu,n+1}. \quad (105)$$

The gauge transformations for n -th index are

$$\Phi_{n,n+1} \rightarrow U_n \Phi_{n,n+1}, \quad (106)$$

$$D_\mu \Phi_{n,n+1} \rightarrow \partial_\mu U_n \Phi_{n,n+1} + U_n \partial_\mu \Phi_{n,n+1} + ig A'_{\mu,n} U_n \Phi_{n,n+1} - ig U_n \Phi_{n,n+1} A_{\mu,n+1}. \quad (107)$$

Since the right hand side of (107) should be $U_n (\partial_\mu \Phi_{n,n+1} + ig A_{\mu,n} \Phi_{n,n+1} - ig \Phi_{n,n+1} A_{\mu,n+1})$ in the correct gauge transformations, the following relation should be satisfied,

$$ig A'_{\mu n} = U_n ig A_{\mu,n} U_n^{-1} - (\partial_\mu U_n) U_n^{-1}. \quad (108)$$

In the same manner, for the $(n+1)$ -th index, we have

$$\Phi_{n,n+1} \rightarrow \Phi_{n,n+1} U_{n+1}^{-1}, \quad (109)$$

$$D_\mu \Phi_{n,n+1} \rightarrow \partial_\mu \Phi_{n,n+1} U_{n+1}^{-1} + \Phi_{n,n+1} \partial_\mu U_{n+1}^{-1} + ig A_{\mu n} \Phi_{n,n+1} U_{n+1}^{-1} - ig \Phi_{n,n+1} U_{n+1}^{-1} A'_{\mu,n+1}. \quad (110)$$

Since the right hand side of (110) should be $(\partial_\mu \Phi_{n,n+1} + ig A_{\mu,n} \Phi_{n,n+1} - ig \Phi_{n,n+1} A_{\mu,n+1}) U_{n+1}^{-1}$, the relations

$$ig A'_{\mu,n+1} = U_{n+1} ig A_{\mu,n+1} U_{n+1}^{-1} - (\partial_\mu U_{n+1}) U_{n+1}^{-1}, \quad (111)$$

are required. Therefore, we find from (108) and (111) that both n and $(n+1)$ -th fields in a bi-fundamental one can be covariant under the same gauge transformation:

$$D_\mu \Phi_{n,n+1} \rightarrow U_n(D_\mu \Phi_{n,n+1})U_{n+1}^{-1}. \quad (112)$$

When we compactify the extra-dimensional space by S^1 meaning $U_N = U_0$, the Lagrangian of $[SU(N_c)]^N$ gauge theory in four dimensions, which is equivalent to $SU(N_c)$ gauge theory in five dimension, is written by

$$\mathcal{L} = \sum_{n=0}^{N-1} |D_\mu \Phi_{n,n+1}|^2 + \text{tr}(\Phi_{0,1} \Phi_{1,2} \cdots \Phi_{N-1,N}). \quad (113)$$

Here, let us consider a VEV, $\langle \Phi_{n,n+1} \rangle = v\delta_{n,n+1}$, and expansion around the VEV as

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1N_c} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N_c1} & A_{N_c2} & \cdots & A_{N_cN_c} \end{pmatrix} \equiv \Phi_{n,n+1} = iv\delta_{n,n+1} + \Phi'_{n,n+1}, \quad (114)$$

where the bi-fundamental field, $\Phi_{n,n+1}$, is transformed by the same U for all n as shown above, $\Phi_{n,n+1} \rightarrow U\Phi_{n,n+1}U^{-1}$. Under this expansion, the kinetic term for the scalar field in (113) can be rewritten by

$$\sum_{n=0}^{N-1} |D_\mu \Phi_{n,n+1}|^2 = \sum_{n=0}^{N-1} |\partial_\mu \Phi'_{n,n+1} - gv(A_{\mu,n} - A_{\mu,n+1}) + igA_{\mu,n}\Phi'_{n,n+1} - ig\Phi'_{n,n+1}A_{\mu,n}|^2. \quad (115)$$

Comparing this mass term of gauge field with that of (103), we find the following correspondences among the gauge coupling, VEV, size of lattice, compactification radius, and number of $SU(N_c)$,

$$gv \leftrightarrow \frac{1}{a}, \quad \pi R \leftrightarrow aN. \quad (116)$$

Let us return to the discussion about mass term of gauge field (103). From the above correspondences (116), it is seen that the mass term of gauge field can be described as

$$\mathcal{L}_{\text{kin}} \supset \sum_{n=0}^{N-1} g^2 v^2 (A_{\mu,n} - A_{\mu,n+1})^2, \quad (117)$$

after the deconstruction.

What about the five-dimensional Lagrangian and extra-dimensional BCs for gauge field under this deconstruction? We start with the following five-dimensional Lagrangian,

$$\mathcal{L}_{5D} = -\frac{1}{2}\text{tr}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\text{tr}(\partial_\mu A_5 - \partial_5 A_\mu + ig[A_\mu, A_5])^2. \quad (118)$$

After latticizing the five-dimensional coordinate, the Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{5D} \rightarrow & -\frac{1}{2} \sum_n^{N-1} \text{tr} F_{\mu\nu}^n F^{n\mu\nu} \\ & -\frac{1}{2} \sum_n^{N-1} \text{tr} \left| \partial_\mu A_{5,n,n+1} - \left(\frac{A_{\mu,n+1} - A_{\mu,n}}{a} \right) + ig(A_{\mu,n} A_{5,n,n+1} - A_{5,n,n+1} A_{\mu,n+1}) \right|^2. \end{aligned} \quad (119)$$

Notice that the gauge field $A_{\mu,n}$ is transformed as $A_{\mu,n} \rightarrow U_n A_{\mu,n} U_n^{-1} - (\partial_\mu U_n) U_n^{-1}$. It is seen that the Dirichlet and Neumann type BCs can be written by

$$A_{\mu,0} = 0, \quad A_{\mu,N} = 0, \quad (120)$$

for Dirichlet BCs at the boundaries, and

$$A_{\mu,0} = A_{\mu,1}, \quad A_{\mu,N-1} = A_{\mu,N}, \quad (121)$$

for Neumann ones, respectively. We comment on the scalar sector described by a bi-fundamental field. The general Lagrangian is given as

$$\mathcal{L} = \sum_{n=1}^{N-1} \text{tr} |D_\mu \Phi_{n,n+1}|^2 + V(x), \quad (122)$$

where

$$x \equiv \text{tr} |\Phi_{0,1} \Phi_{1,2} \cdots \Phi_{N-1,N}|^2, \quad V(x) = \lambda(x^2 - v^{2N})^2. \quad (123)$$

The BCs for this bi-fundamental field are

$$\Phi_{0,1} = 0, \quad \Phi_{N-1,N} = 0, \quad (124)$$

for Dirichlet BCs, and

$$\Phi_{0,1} = \Phi_{1,2}, \quad \Phi_{N-2,N-1} = \Phi_{N-1,N}, \quad (125)$$

for Neumann ones, respectively.

So far, we obtained the descriptions of mass term of gauge boson (117) and BCs (120) and (121). Finally, we show typical wave-function profiles of gauge field under the above each BC. Under the Neumann type BCs (121), the mass term of gauge boson can be written down by

$$g^2 v^2 (A_{\mu,1} \cdots A_{\mu,N}) \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 & 0 & 0 \\ -1 & 2 & -1 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2 & -1 & 0 \\ 0 & 0 & 0 & \cdots & -1 & 2 & -1 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} A_{\mu,1} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ A_{\mu,N} \end{pmatrix}, \quad (126)$$

where we consider the sum up to N . For the Dirichlet BC, the mass term is described as

$$g^2 v^2 (A_{\mu,1} \cdots A_{\mu,N}) \begin{pmatrix} 2 & -1 & 0 & \cdots & 0 & 0 & 0 \\ -1 & 2 & -1 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2 & -1 & 0 \\ 0 & 0 & 0 & \cdots & -1 & 2 & -1 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} A_{\mu,1} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ A_{\mu,N} \end{pmatrix}. \quad (127)$$

In the deconstruction method, one can obtain gauge field profiles in an extra-dimensional direction by taking the wave-function in the basis of mass eigenstate. The basis of eigenstate are changed by operating an unitary matrix, U_A , which diagonalizes the mass matrix given in (126) or (127), as

$$g^2 v^2 (A_{\mu,1} \cdots A_{\mu,N}) U_A^\dagger \begin{pmatrix} m_{A^{(0)}}^2 & 0 & \cdots & 0 \\ 0 & m_{A^{(1)}}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_{A^{(N-1)}}^2 \end{pmatrix} U_A \begin{pmatrix} A_{\mu,1} \\ \vdots \\ \vdots \\ A_{\mu,N} \end{pmatrix}. \quad (128)$$

Therefore, the mass eigenstate for each KK mode can be described by

$$\begin{pmatrix} A_\mu^{(0)} \\ A_\mu^{(1)} \\ \vdots \\ A_\mu^{(N-1)} \end{pmatrix} = U_A \begin{pmatrix} A_{\mu,1} \\ A_{\mu,2} \\ \vdots \\ A_{\mu,N} \end{pmatrix} \quad (129)$$

$$= \begin{pmatrix} (U_A)_{11} A_\mu(a) + (U_A)_{12} A_\mu(2a) + \cdots + (U_A)_{1N} A_\mu(Na) \\ (U_A)_{21} A_\mu(a) + (U_A)_{22} A_\mu(2a) + \cdots + (U_A)_{2N} A_\mu(Na) \\ \vdots \\ (U_A)_{N1} A_\mu(a) + (U_A)_{N2} A_\mu(2a) + \cdots + (U_A)_{NN} A_\mu(Na) \end{pmatrix}. \quad (130)$$

Since this description is one after the dimensional reduction of extra-dimension, the wave-function profile for the field $A_\mu^{(n)}$ is composed of $(a, U_{n1}), (2a, U_{n2}), \cdots, (Na, U_{nN})$. The numerical plots of the wave-function profile for $N = 30$ case are shown in Fig. 2.

Finally, we present an example in the warped case¹³. The mass matrices of gauge boson in the Neumann (126) and Dirichlet (127) are modified to

$$\begin{pmatrix} e^{-2ka} & -e^{-2ka} & \cdots & 0 & 0 \\ -e^{-2ka} & e^{-3ka} F(ka) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & e^{-(2(N-1)-1)ka} F(ka) & -e^{-2(N-1)ka} \\ 0 & 0 & \cdots & -e^{-2(N-1)ka} & e^{-2(N-1)ka} \end{pmatrix}, \quad (131)$$

¹³See ref. [55] for deconstructing gauge theories in AdS₅.

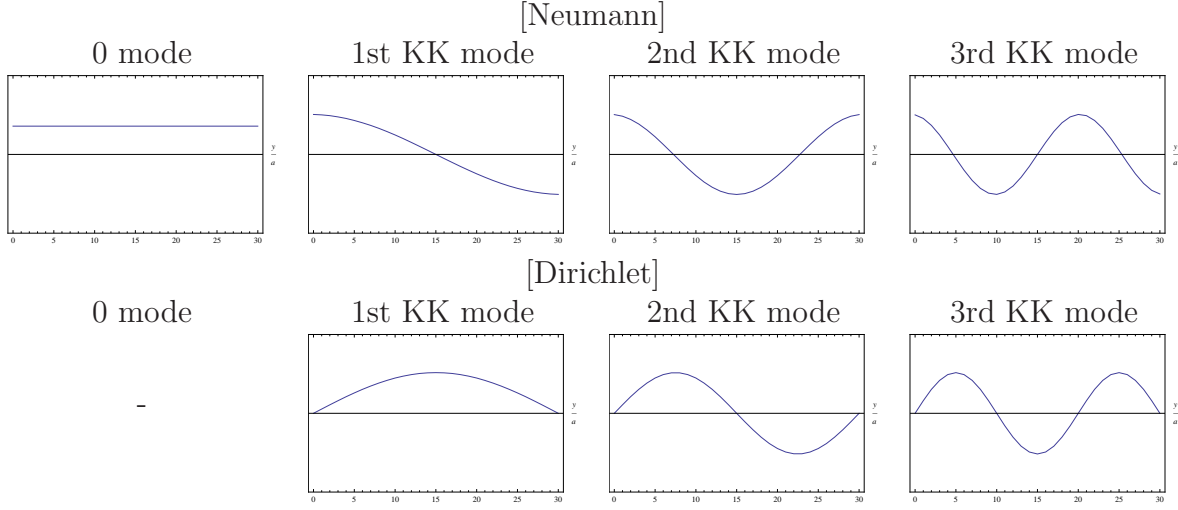


Figure 2: The numerical plots of the gauge field profiles for $N = 30$ case in the deconstruction method for the flat extra-dimension. In the Dirichlet BC case, there is not a zero mode.

and

$$\begin{pmatrix} e^{-ka}F(ka) & -e^{-2ka} & \dots & 0 & 0 \\ -e^{-2ka} & e^{-3ka}F(ka) & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & e^{-(2(N-1)-1)ka}F(ka) & -e^{-2(N-1)ka} \\ 0 & 0 & \dots & -e^{-2(N-1)ka} & e^{-2(N-1)ka}F(ka) \end{pmatrix}, \quad (132)$$

respectively, where $F(ka) \equiv e^{-ka} + e^{ka}$. The numerical plots of the wave-function profile for $N = 30$ case are shown in Fig. 3.

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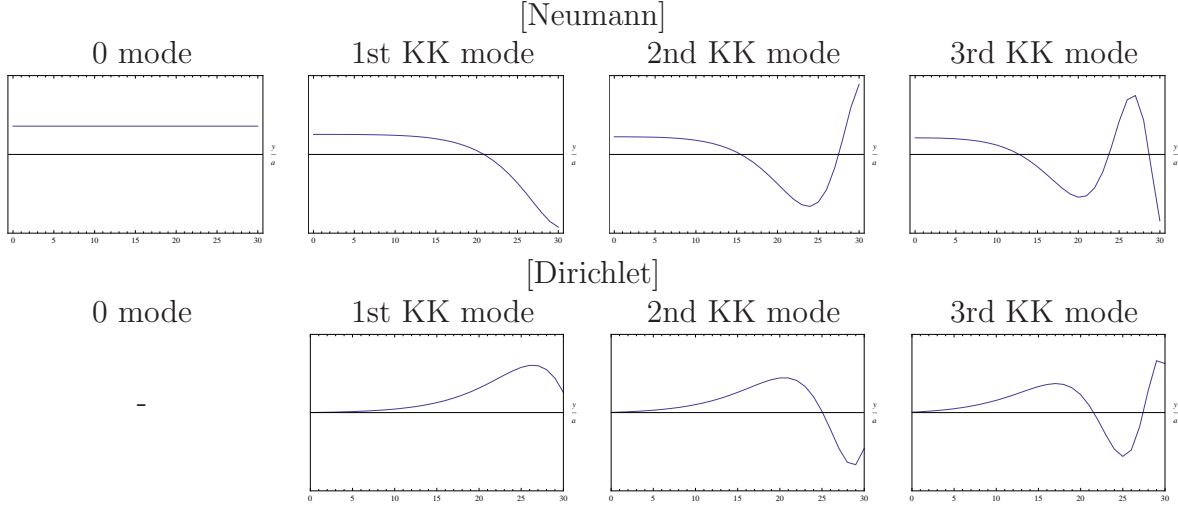


Figure 3: The numerical plots of the gauge field profiles for $N = 30$ case in the deconstruction method for the warped extra-dimension. In the Dirichlet BC case, there is not a zero mode.

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